Activeness and Responsiveness in Mobile Processes

Maxime Gamboni

Instituto de Telecomunicações, Instituto Superior Técnico

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Joint work with António Ravara
This Work

*Characterising two liveness properties in a mobile process* through the use of a *type system*. 
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*Characterising* two *liveness properties* in a *mobile process* through the use of a *type system*.
Type System

- Types: descriptions
- Type Semantics: formal meaning
- Type System: computable algorithm
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Soundness and Completeness

- Soundness
- Completeness
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Soundness and Completeness

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This Work

Characterising two liveness properties in a mobile process through the use of a type system.
The Synchronous Polyadic $\pi$-calculus

$$P = ((\nu x) \bar{a}(x).x(y).\bar{y}(42)) \parallel !a(r).\bar{r}(s)$$

- Message sending
- Independent execution
- Private channel
- Receiving messages
- Forking
The Synchronous Polyadic $\pi$-calculus

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Message sending

Independent execution

Forking

Receiving messages

Private channel
The Synchronous Polyadic $\pi$-calculus

$$P = ((\nu x)\overline{a}(x) . x(y) . \overline{y}(42)) | !a(r) . \overline{r}s$$

Message sending

Private channel

Receiving messages

Forking

Independent execution
The Synchronous Polyadic $\pi$-calculus

$$P = \left( (\nu x) \overline{a}(x).x(y).\overline{y}(42) \right) \mid !a(r).\overline{r}(s)$$

- Private channel
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The Synchronous Polyadic $\pi$-calculus

$P = (((\nu x) \overline{a}(x).x(y).\overline{y}(42)) | !a(r).\overline{r}\langle s\rangle)$

- Message sending
- Independent execution
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- Forking
Transitions

\[
((\nu x) \overline{a}(x).x(y).\overline{y}(42)) \mid a(r).\overline{r}(s)
\]

\[
\rightarrow (\nu x) (x(y).\overline{y}(42) \mid \overline{x}(s))
\]

\[
\rightarrow (\nu x) (\overline{s}(42) \mid 0)
\]

\[
\overline{s}(42) \quad \rightarrow \quad (\nu x) (0 \mid 0)
\]

\[
\equiv 0
\]
Transitions

\[
\begin{align*}
(\nu x) (a(x) . y(42) ) & \rightarrow (\nu x) . y(42) \\
(\nu x) . \bar{r}(s) & \rightarrow (\nu x) (\bar{s}(42) | 0) \\
(\nu x) (\bar{s}(42) | 0) & \rightarrow (\nu x) (0 | 0) \\
& \equiv 0
\end{align*}
\]
Transitions

\[
\begin{align*}
((\nu x) \overline{a} \langle x \rangle . x(y). \overline{y} \langle 42 \rangle) & | a(r). \overline{r} \langle s \rangle \\
\rightarrow & (\nu x)(x(y). \overline{y} \langle 42 \rangle | \overline{x} \langle s \rangle) \\
\rightarrow & (\nu x)(\overline{s} \langle 42 \rangle | 0) \\
\overline{s} \langle 42 \rangle & \rightarrow (\nu x)(0 | 0) \\
\equiv & 0
\end{align*}
\]
Transitions

\[
((\nu x) \overline{a}\langle x \rangle . x(y) . \overline{y}\langle 42 \rangle) \mid a(r) . \overline{r}\langle s \rangle
\]

\[
\rightarrow (\nu x) (x(y) . \overline{y}\langle 42 \rangle \mid \overline{x}\langle s \rangle)
\]

\[
\rightarrow (\nu x) (\overline{s}\langle 42 \rangle \mid 0)
\]

\[
\overline{s}\langle 42 \rangle \rightarrow (\nu x) (0 \mid 0)
\]

\[
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\]
Transitions

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((\nu x) \overline{a} \langle x \rangle . x (y) . \overline{y} \langle 42 \rangle) \mid a(r) . \overline{r} \langle s \rangle \\
\rightarrow (\nu x) (x (y) . \overline{y} \langle 42 \rangle \mid \overline{x} \langle s \rangle) \\
\rightarrow (\nu x) (\overline{s} \langle 42 \rangle \mid 0) \\
\overline{s} \langle 42 \rangle \\
\overrightarrow{0} (\nu x) (0 \mid 0) \\
\equiv 0
\]
This Work

*Characterising two liveness properties in a mobile process through the use of a type system.*
Activeness

**Definition (Activeness)**

Activeness $p_A$ of a port $p \in \{a, \overline{a}\}$ in a process $P$: Ability of $P$ to reliably receive ($p = a$) or send ($p = \overline{a}$) a message on it.

- $\overline{s}$ active in $((\nu x) \overline{a} \langle x \rangle . x(y) . \overline{y} \langle 42 \rangle) | a(r) . \overline{r} \langle s \rangle$,
- $\overline{s}$ not active in $((\nu x) \overline{a} \langle x \rangle . x(y) . \overline{y} \langle 42 \rangle) | a(r) . \overline{r} \langle s \rangle | \overline{a} \langle w \rangle$. 
Activeness

Definition (Activeness)

Activeness $p_A$ of a port $p \in \{a, \bar{a}\}$ in a process $P$: Ability of $P$ to reliably receive ($p = a$) or send ($p = \bar{a}$) a message on it.

- $s$ active in $((\nu x) \bar{a}(x).x(y).\bar{y}(42)) | a(r).\bar{r}(s)$,
- $\bar{s}$ not active in $((\nu x) a(x).x(y).\bar{y}(42)) | a(r).\bar{r}(s) | \bar{a}(w)$.
Activeness

Definition (Activeness)

Activeness $p_A$ of a port $p \in \{a, \bar{a}\}$ in a process $P$: Ability of $P$ to reliably receive ($p = a$) or send ($p = \bar{a}$) a message on it.

- $\bar{s}$ active in $((\nu x) \bar{a}\langle x\rangle . x(y) . \bar{y}\langle 42\rangle) \mid a(r) . \bar{r}\langle s\rangle$,
- $\bar{s}$ not active in $((\nu x) \bar{a}\langle x\rangle . x(y) . \bar{y}\langle 42\rangle) \mid a(r) . \bar{r}\langle s\rangle \mid \bar{a}\langle w\rangle$. 
Responsiveness

Definition (Responsiveness)

*Responsiveness* $p_R$ of a port $p$ in a process $P$ is the ability, every time a communication occurs on that port, to continue the conversation as far as requested by the other party.

- Port $\overline{a}$ is responsive but not active in $?\overline{a}\langle x\rangle.x(y).\overline{y}\langle 42\rangle$
- Port $\overline{a}$ is active but not responsive in $\overline{a}\langle x\rangle.x(y).?\overline{y}\langle 42\rangle$
Responsiveness

Definition (Responsiveness)

*Responsiveness* $p_R$ of a port $p$ in a process $P$ is the ability, every time a communication occurs on that port, to continue the conversation as far as requested by the other party.

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This Work

`Characterising` two liveness properties in a mobile process through the use of a type system.
Environment and Composition

\[(L; E) \circ (L'; E') \overset{\text{def}}{=} (L \circ L'; (E \setminus L') \diamond (E' \setminus L))\]
Conditional Activeness

\[(\overline{t}.a \mid \overline{u}.a) \mid (\overline{v}.\overline{a}.\overline{w}.\overline{s}) \mid (u \mid w)\]

- $\overline{s}$ depends on $v$, $a$ and $w$
- $a$ depends on any one of $t$ or $u$
- and $u$, $w$ are provided on the right
- Therefore $\overline{s}$ only depends on $v$
Conditional Activeness

\[
(t.a | \bar{u}.a) \mid (\bar{v}.\bar{a}.\bar{w}.\bar{s}) \mid (u \mid w)
\]

- \(\bar{s}\) depends on \(v\), \(a\) and \(w\)
- \(a\) depends on any one of \(t\) or \(u\)
- and \(u\), \(w\) are provided on the right
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Conditional Activeness

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- \(\bar{s}\) depends on \(v\), \(a\) and \(w\)
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- and \(u\), \(w\) are provided on the right
- Therefore \(\bar{s}\) only depends on \(v\)
Labelled Dependencies

\[ P = (\nu t) (\bar{t} \mid t.(\bar{z}.a(x).\bar{z}.\bar{x}) \mid t.a(y).\bar{y}) \]

\[ \bar{t}_A \triangleleft \bot ; \quad z_A \triangleleft \bar{t}_A ; \quad a_R \triangleleft z_A \quad \Rightarrow \quad a_R \triangleleft \bot \]

**Labelled Dependencies**

- **Labels l, l', ...**
- \( \bar{l} \lor \varepsilon \): Only need \( \varepsilon \) if “l” occurred.
- \( l \lor \varepsilon \): Need \( \varepsilon \) unless “l” occurred.

Set “l” to “The left t.-prefix got consumed”.

\[ \bar{t}_A \triangleleft \bot ; \quad z_A \triangleleft l \lor \bar{t}_A ; \quad a_R \triangleleft \bar{l} \lor z_A \quad \Rightarrow \quad a_R \triangleleft \top \]
Multiplicities

\[ P = ((\nu x)\, \overline{a}(x) \cdot x(y) \cdot \overline{y}(42)) \mid ! a(r) \cdot \overline{r}(s) \]

- \( \overline{a}^1 \): One output on \( a \)
- \( a^\omega \): Arbitrarily many inputs on \( a \)
- \( \overline{s}^\star \): Unspecified number of outputs on \( s \)
Multiplicities

\[ P = ((\nu x) \overline{a}(x).x(y).\overline{y}(42)) \mid !a(r).\overline{r}(s) \]

- \( \overline{a}^1 \): One output on \( a \)
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Multiplicities

\[ P = ((\nu x) \bar{a}(x).x(y).\bar{y}(42)) | ! a(r).\bar{r}(s) \]

- \( \bar{a}^1 \): One output on \( a \)
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Composing Multiplicity Types

\[(c^\omega; \bar{c}^\star) \vdash \! c(tu).\bar{t}\langle u \rangle\]
\[(\bar{a}c\bar{c}x; c^\omega \bar{c}^\star a^\omega x) \vdash \bar{c}\langle ax \rangle\]
\[\left\{\begin{array}{l}
(\bar{a}c\bar{c}x c^\omega a^\omega \bar{s}^\star; \bar{c}^\star s^\omega x) \vdash \bar{c}\langle ax \rangle | \! c(tu).\bar{t}\langle u \rangle | \! a(r).\bar{r}\langle s \rangle
\end{array}\right\}

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Composing Multiplicity Types

\[
\begin{align*}
(c^\omega; \bar c^\star) & \vdash !c(tu).\bar t\langle u \rangle \\
(a\bar c\bar x; c^\omega \bar c^\star a^\omega x) & \vdash \bar c\langle ax \rangle \\
(a\bar c\bar x c^\omega a^\omega \bar s^\star; \bar c^\star s^\omega x) & \vdash \bar c\langle ax \rangle | !c(tu).\bar t\langle u \rangle | !a(r).\bar r\langle s \rangle
\end{align*}
\]
Composing Multiplicity Types

\[
\begin{align*}
(c^\omega; \bar{c}^\star) & \vdash !c(tu).\bar{t}\langle u \rangle \\
(a\bar{c}\bar{x}; c^\omega \bar{c}^\star a^\omega x) & \vdash \bar{c}\langle ax \rangle \\
(a\bar{c}\bar{x}c^\omega a^\omega \bar{s}^\star; \bar{c}^\star s^\omega x) & \vdash \bar{c}\langle ax \rangle | !c(tu).\bar{t}\langle u \rangle | !a(r).\bar{r}\langle s \rangle
\end{align*}
\]
Composing Multiplicity Types

\[(c^\omega; \bar{c}\star) \vdash !c(tu).t\langle u \rangle\]
\[(\bar{a}\bar{c}\bar{x}; c^\omega \bar{c}\star a^\omega x) \vdash \bar{c}\langle ax \rangle !c(tu).t\langle u \rangle\]

\[(\bar{a}\bar{c}\bar{x} c^\omega a^\omega s\star; \bar{c}\star s^\omega x) \vdash \bar{c}\langle ax \rangle !c(tu).t\langle u \rangle !a(r).\bar{r}\langle s \rangle\]
Conclusion

*Characterising two liveness properties in a mobile process through the use of a type system.*

Our contribution:
- A formalism describing liveness properties in the $\pi$-calculus
- Environment in the type $\Rightarrow$ Compositionality
- Labels $\Rightarrow$ non-transitive dependencies

More info:
- [http://maxime.gamboni.org/](http://maxime.gamboni.org/)