

# Responsive Choice in Mobile Processes

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<sup>1</sup>Joint work with António Ravara

# Outline

## Goal

- Verifying Calculus Encodings
- Using Choice and Branching to encode Data

$$\llbracket \text{True} \rrbracket_a = ! a(tf).\bar{t} \quad \llbracket \text{False} \rrbracket_a = ! a(tf).\bar{f}$$

$$\llbracket \text{If } (e) \text{ Then } (T) \text{ Else } (F) \rrbracket = (\nu btf) (\llbracket e \rrbracket_b \mid \bar{b}\langle tf \rangle \mid (t.\llbracket T \rrbracket + f.\llbracket F \rrbracket))$$

## Contribution

- Activeness and Responsiveness
- Choice Types
- Conditional Dependencies
- Sound Type System

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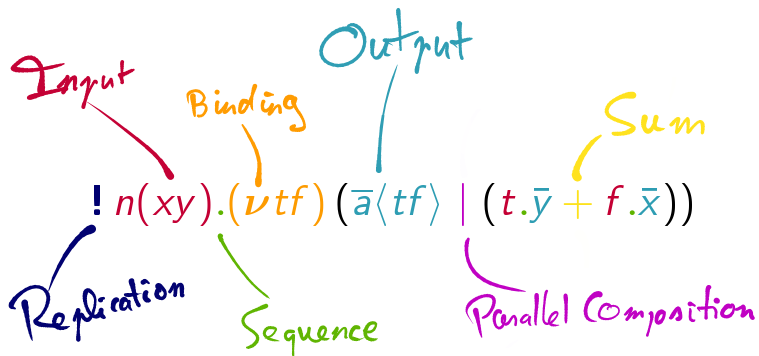
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# The Synchronous Polyadic $\pi$ -calculus



# Outline

## Contribution

- **Activeness and Responsiveness**
- Choice Types
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# Responsiveness

## Definition (**A**=Activeness)

- $a_A$ : I am soon ready to receive at  $a$
- $\bar{a}_A$ : I am soon ready to send to  $a$

$$\bar{y}_A \models \bar{t} \mid (t.\bar{y} + f.\bar{x})$$



$$\bar{y}_A \not\models \bar{t} \mid (t.\bar{y} + t.\bar{x})$$





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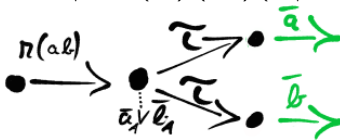


# Responsiveness

## Definition (**R**=Responsiveness)

- $a_R$ : If I get a message from you at  $a$ , I'll obey  $a$ 's protocol
- $\bar{a}_R$ : If you get a message from me at  $a$ , I'll obey  $\bar{a}$ 's protocol

$$r : \text{Bool}, r_R \models !r(tf).(\nu q)(\bar{q} \mid q.\bar{t} \mid q.\bar{f})$$



$$a : \text{Bool}, \bar{a}_R \models (\bar{a}\langle tf \rangle \mid (t.\bar{y} + f.\bar{x}))$$

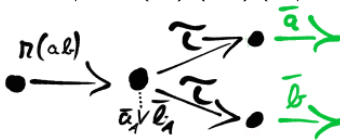


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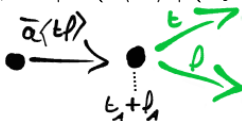
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# Multiplicities

## Definition (Multiplicity)

$p^m$ : I'll use  $p$  *at most*  $m$  times.

- $p^0$ : Never
- $p^1$ : Once
- $p^\omega$ : Once, replicated
- $p^\star$ : Many times

# Outline

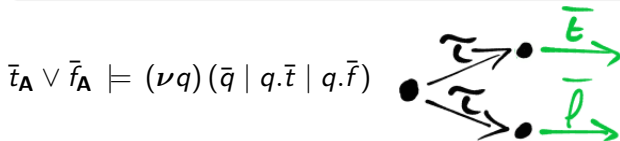
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- Choice Types
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- Sound Type System

# Choice

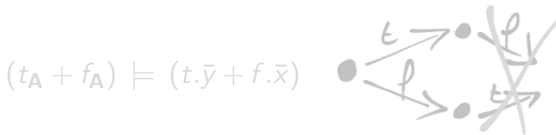
## Definition (Choice $A \vee B$ )

I will either behave like  $A$  or like  $B$



## Definition (Branching $A + B$ )

You can make me do  $A$  or  $B$  (not both)

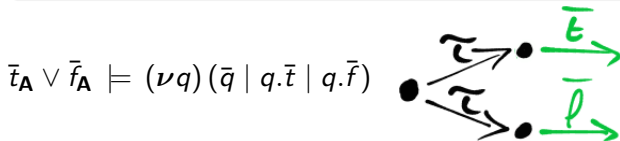




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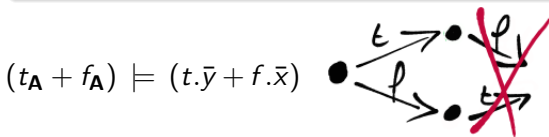
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# Dependencies

## Definition (Dependency $A \triangleleft B$ )

If you give me  $B$ , I'll give you  $A$ .

$$n_R \triangleleft (a_A \wedge a_R) \models !n(xy).\bar{a}(\nu tf).(t.\bar{y} + f.\bar{x})$$



# Types

## Definition (Channel Type)

(parameters ; input ; output)

- $\text{Bool} \stackrel{\text{def}}{=} (()) ; \bar{1}_{\mathbf{A}} \vee \bar{2}_{\mathbf{A}} ; 1_{\mathbf{A}} + 2_{\mathbf{A}}$
- $!a(xy).(\nu q)(\bar{q}|q.\bar{x}|q.\bar{y}) \mid \bar{a}\langle \text{tf} \rangle \mid (t.P + f.Q)$

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# Typing — Composition

$$(\bar{t}_A \vee \bar{f}_A) \triangleleft a_{AR} \vdash \bar{a} \langle tf \rangle$$



$$((t_A, \bar{y}_A \triangleleft \bar{t}_A) + (f_A, \bar{x}_A \triangleleft \bar{f}_A)) \vdash (t.\bar{y} + f.\bar{x})$$



$$(\bar{t}_A \vee \bar{f}_A) \triangleleft a_{AR}, ((t_A, \bar{y}_A \triangleleft \bar{t}_A) + (f_A, \bar{x}_A \triangleleft \bar{f}_A)) \vdash \bar{a} \langle tf \rangle \mid (t.\bar{y} + f.\bar{x})$$

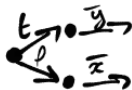


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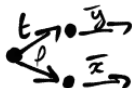
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# Typing — $\vee$ -Distributivity

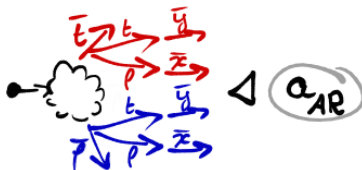
## $\vee$ -Distributivity

“ $\vee$  distributes with everything else”

$$(\bar{t}_A \vee \bar{f}_A) \triangleleft a_{AR}, ((t_A, \bar{y}_A \triangleleft \bar{t}_A) + (f_A, \bar{x}_A \triangleleft \bar{f}_A)) =$$

$$(\bar{t}_A \triangleleft a_{AR}, ((t_A, \bar{y}_A \triangleleft \bar{t}_A) + (f_A, \bar{x}_A \triangleleft \bar{f}_A))) \vee$$

$$(\bar{f}_A \triangleleft a_{AR}, ((t_A, \bar{y}_A \triangleleft \bar{t}_A) + (f_A, \bar{x}_A \triangleleft \bar{f}_A)))$$



# Typing — Collapse (1)

## Branching Collapse

$$(\bar{p}_A, (p_A + \varepsilon)) \hookrightarrow (\bar{p}_A \wedge p_A) \vee (\bar{p}_A, (p_A + \varepsilon))$$

$$(\bar{t}_A \triangleleft a_{AR}, ((t_A, \bar{y}_A \triangleleft \bar{t}_A) + (f_A, \bar{x}_A \triangleleft \bar{f}_A))) \vee$$

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$$(\bar{t}_A, t_A, (\bar{y}_A \triangleleft \bar{t}_A)) \triangleleft a_{AR} \vee (\bar{f}_A, f_A, (\bar{x}_A \triangleleft \bar{f}_A)) \triangleleft a_{AR}$$



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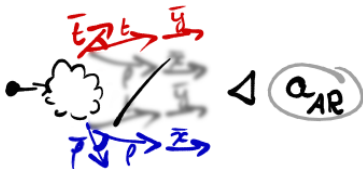
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# Typing — Collapse (2)

## Dependency Collapse

$$(\alpha \triangleleft \beta) \wedge (\beta \triangleleft \gamma) \quad \hookrightarrow \quad (\alpha \triangleleft \beta \vee \gamma) \wedge (\beta \triangleleft \gamma)$$

$$\begin{aligned} & ((t_A \wedge \bar{t}_A) \triangleleft a_{AR}, (\bar{y}_A \triangleleft \bar{t}_A)) \quad \vee \quad ((f_A \wedge \bar{f}_A) \triangleleft a_{AR}, (\bar{x}_A \triangleleft \bar{f}_A)) \quad \hookrightarrow \\ & ((t_A, \bar{t}_A, \bar{y}_A) \triangleleft a_{AR}) \quad \vee \quad ((f_A, \bar{x}_A, \bar{f}_A) \triangleleft a_{AR}) = \end{aligned}$$



$$((t_A, \bar{t}_A, \bar{y}_A) \quad \vee \quad (f_A, \bar{x}_A, \bar{f}_A)) \triangleleft a_{AR} \vdash \bar{a}(tf) \mid (t.\bar{y} + f.\bar{x})$$

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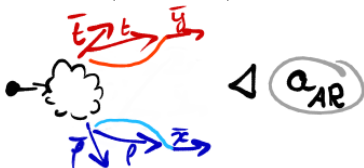


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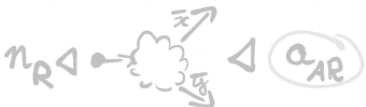
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$$n_{\mathbf{R}} \triangleleft \dots \vdash P = !n(xy).(\nu tf)(\bar{a}\langle tf \rangle \mid (t.\bar{y} + f.\bar{x}))$$

## Responsiveness

$$(\nu \tilde{y}) (n : \sigma \odot n_{\mathbf{R}} \triangleleft \sigma \{\tilde{y}/1\dots n\} \odot \dots) \vdash n(\tilde{y})$$

$$(\nu xy) (n_{\mathbf{R}} \triangleleft (\bar{x}_A \vee \bar{y}_A) \wedge (\bar{x}_A \vee \bar{y}_A) \triangleleft a_{\mathbf{AR}}) \vdash P$$



$$\Leftrightarrow n_{\mathbf{R}} \triangleleft a_{\mathbf{AR}} \vdash !n(xy).(\nu tf)(\bar{a}\langle tf \rangle \mid (t.\bar{y} + f.\bar{x}))$$



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$$\hookrightarrow n_{\mathbf{R}} \triangleleft a_{\mathbf{AR}} \vdash !n(xy).(\nu tf)(\bar{a}\langle tf \rangle \mid (t.\bar{y} + f.\bar{x}))$$



# Conclusion

Our contribution:

- A formalism describing liveness properties in the  $\pi$ -calculus
- Choice and Branching types  $\Rightarrow$  Data encodings
- Conditional Dependencies  $\Rightarrow$  Compositionality

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# Thank you!



More info:

- <http://maxime.gamboni.org/>