Statically Proving Behavioural Properties in the π-calculus via Dependency Analysis

PhD Defence

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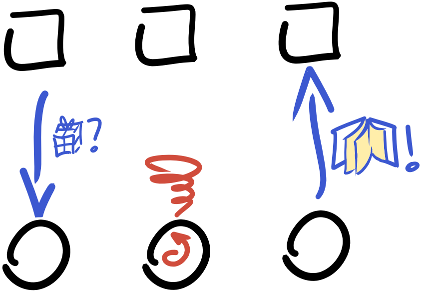
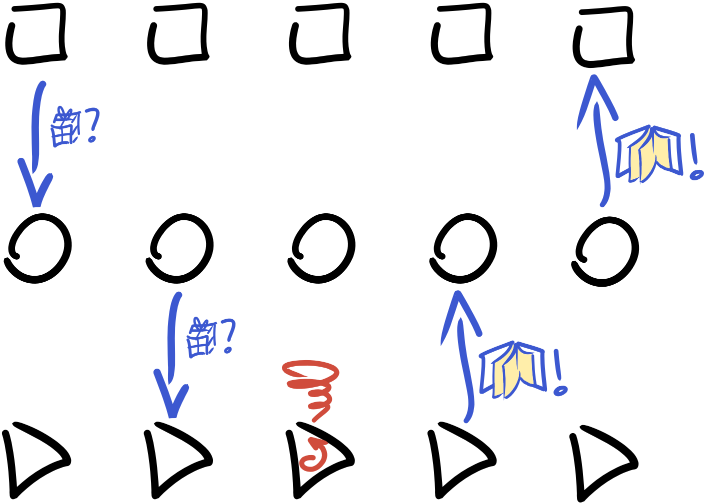
Plan

- Statically Proving
- Behavioural Properties
- in the π-calculus
- via Dependency Analysis

Context: Request & Answer



Context: Proxy



Statical vs Dynamical Analysis

Statically Proving Behavioural Properties in the π-calculus via Dependency Analysis

Definition (Model Checking)

Finding Properties by simulating execution

Definition (Statical Analysis)

Finding Properties without running the program

Statical vs Dynamical Analysis

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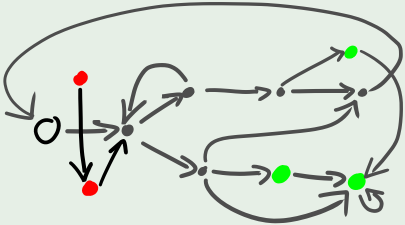
Model Checking

Finding/Verifying properties by simulating execution

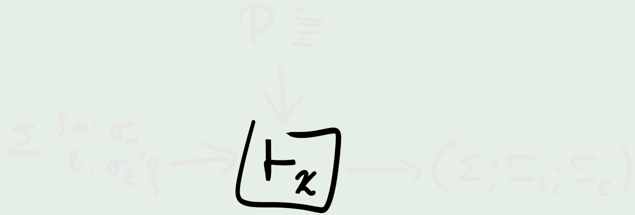
Model Checking



Type Systems

Finding/Verifying properties without running the program

My Type Inference System



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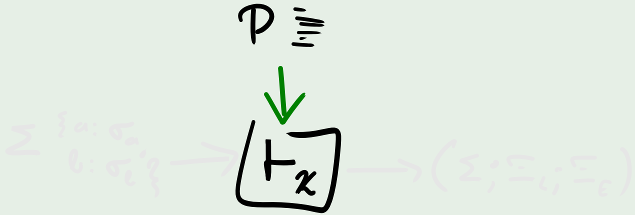

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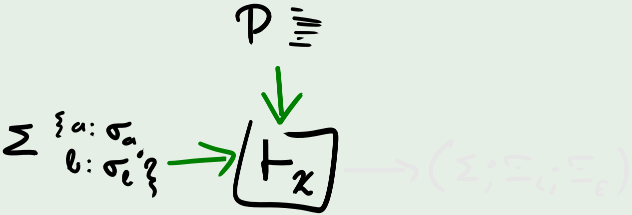

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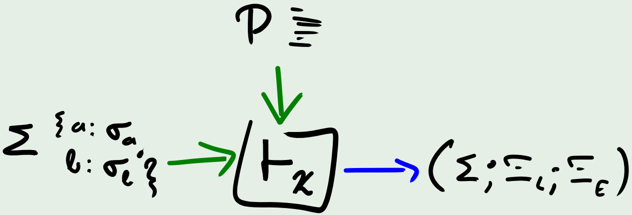

Behavioural Properties

Statically Proving Behavioural Properties in the π-calculus via Dependency Analysis

Examples

- Activeness (Receptiveness)
- Isolation

Behavioural Properties

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Behavioural Properties: Existential vs Universal

Definition (Existential Property)

Available *somewhere*. Good things happen *eventually*.

e.g. "Activeness"

Definition (Universal Property)

Available *everywhere*. Good things happen *constantly*.

e.g. "Isolation"

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The π-calculus

Statically Proving Behavioural Properties in the π-calculus via Dependency Analysis

The π-calculus

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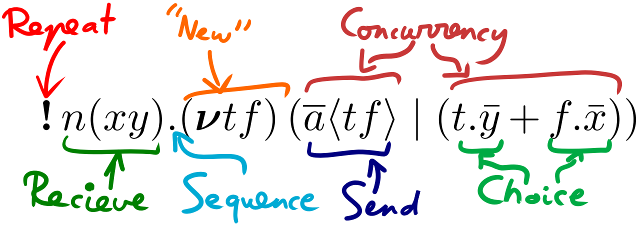
$$!\,n(xy).(\boldsymbol{\nu} t f)\,(\overline{a}\langle t f\rangle \mid (t.\bar{y} + f.\bar{x}))$$



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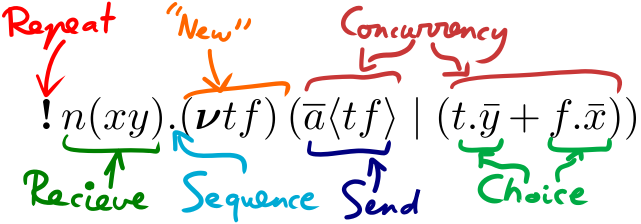

$$!\,n(xy).(\boldsymbol{\nu} t f)\,(\overline{a}\langle t f\rangle \mid (t.\bar{y} + f.\bar{x}))$$

Example

$$\bigcirc(qr).\overline{\nabla}\langle qr'\rangle.r'(a).\overline{r}\langle a\rangle$$

Dependency Analysis

Statically Proving Behavioural Properties in the π-calculus via Dependency Analysis

Definition (Dependency $A \lhd B$)

If you give me B, I'll give you A.

○ *is* **I***solated if* ▽ *is* **I***solated*

$$(\bigcirc_{\mathbf{I}}) \lhd (\triangledown_{\mathbf{I}})$$

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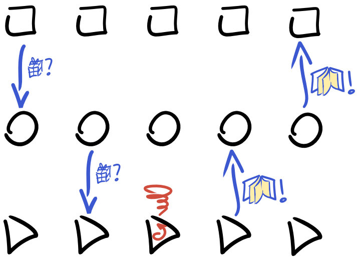

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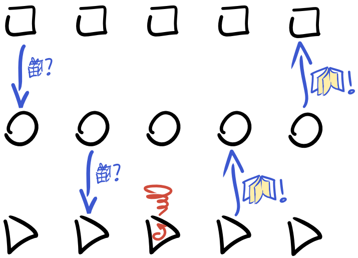

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$$(\bigcirc_{\mathbf{I}}) \lhd (\triangledown_{\mathbf{I}})$$

Generic Type System

- Not specific to a property

Instantiation:

- Write *semantic goals*
- Rules *parametrised* by *elementary rules*

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Contributions

Type Language		Process Behaviour
Selection & Branching	$A \vee B$, $p + q$	Choice
Activeness	$p_{\mathbf{A}}$	Liveness
Determinism, Isolation, ...	$p_{\mathbf{D}}$, $p_{\mathbf{I}}$, ...	Safety
Dependencies	$A \lhd B$	Causality

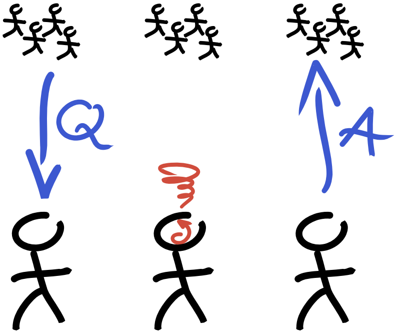
Generic Type System:

- Decidable
- Constructs Logical Formulæ
- Sound
- Compositional

Conclusion

"Statically Proving Behavioural Properties in the π-calculus via Dependency Analysis"

Questions



Supplementary Material

- Types & Multiplicities
- Choice
- Algebra
- Semantics
- Type Systems
- Properties
- Soundness
- Future Work

Types & Multiplicities

Behavioural Statements Δ, Ξ, ...

$$\Delta \quad ::= \quad \Delta \vee \Delta \;\mid\; \Delta + \Delta \;\mid\; \Delta \wedge \Delta \;\mid\; \Delta \lhd \Delta \;\mid\; p_k \;\mid\; \bot \;\mid\; \top \;\mid\; p^m$$

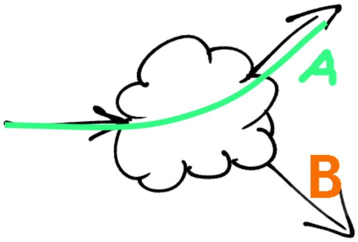
Multiplicities

$$m \quad ::= \quad 0 \;\mid\; 1 \;\mid\; \omega \;\mid\; \star$$

Choice

Definition (Selection $A \vee B$)

I will either behave like A or like B



Definition (Branching $A + B$)

You can make me do A or B

Choice

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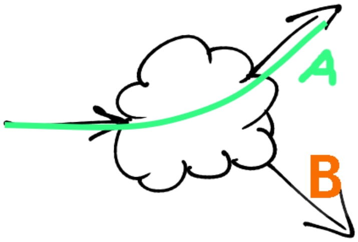

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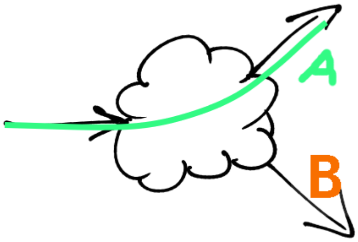


Definition (Branching $A + B$)

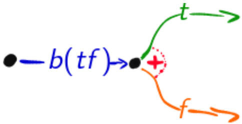
You can make me do A or B

Choice Examples (I)

- Data Encodings

$$b := \text{True} \stackrel{\text{def}}{=} !\, b(tf).\overline{t}$$

$$b := \text{False} \stackrel{\text{def}}{=} !\, b(tf).\overline{f}$$

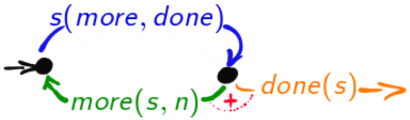


$$\text{If } b \text{ Then } P \text{ Else } Q \stackrel{\text{def}}{=} \overline{b}(\nu tf).(t.P+f.Q)$$

Choice Examples (II)

- Client-Server Conversations

$$\overline{prod}(\boldsymbol{\nu} s).s(more, done).\overline{more}(\boldsymbol{\nu} s, 2).$$
$$s(more, done).\overline{more}(\boldsymbol{\nu} s, 5).$$
$$s(more, done).\overline{done}(\boldsymbol{\nu} s).s(x).\overline{print}\langle x\rangle$$



$$!\,prod(s).\overline{p_0}\langle 1, s\rangle \quad | \quad !\,p_0(t, s).\overline{s}(\boldsymbol{\nu} more, done).$$
$$\big(more(s, n).\overline{p_0}\langle t \times n, s\rangle + done(s).\overline{s}\langle r\rangle\big)$$

Algebra

Spatial Operators

Parallel Composition $\Gamma_1 \odot \Gamma_2$, Restriction $(\boldsymbol{\nu} x)\,\Gamma$, ...

Logical Operators

Equivalence $\cong$, Weakening $\preceq$, Reduction $\hookrightarrow$, ...

Dynamic Operator

Transition Operator $\Gamma \xrightarrow{\mu} (\Gamma \wr \mu)$.

Semantics (Universal)

Definition (Universal Semantics)

A $(\Gamma; P)$ typed process is *correct wrt. universal semantics* ("$\Gamma \models_{\mathcal{U}} P$") if, for all transition sequences $(\Gamma; P) \xrightarrow{\tilde{\mu}}\searrow (\Gamma'; P')$, the local component of Γ' being $\bigvee_{i \in I} p_{i\,k_i} \lhd \varepsilon_i$: for all $i \in I$ with $k_i \in \mathcal{U}$, $\text{good}_{k_i}(p_i \lhd \varepsilon_i, (\Gamma'; P'))$ holds.

Semantics (Existential)

(Abbreviated) Existential Semantics

A typed process $(\Gamma; P)$ is *correct* ("$\Gamma \models P$"), if $\exists$ a strategy f s.t. For any sequence

$(\Gamma; P) = (\Gamma_0; P_0) \cdots \xrightarrow{\tilde{\mu}_i}\searrow (\Gamma'_i; P'_i) \xrightarrow{f} (\Gamma_{i+1}; P_{i+1}) \cdots$, let (for all i) μ_i be the label of $(\Gamma'_i; P'_i) \xrightarrow{f} (\Gamma_{i+1}; P_{i+1})$.

Then $\exists$ a resource p_k and $n \geqslant 0$ such that:

1. $\forall i : (p_k \lhd \mathrm{dep}_{\mathcal{K}}(\mu_i)) \preceq \Gamma'_i$
2. $\exists \varepsilon : (p_k \lhd \varepsilon) \preceq \Gamma_n$ and $\mathrm{good}_k(p \lhd \varepsilon, (\Gamma_n; P_n))$.

Type System (Universal)

$$\frac{\forall i : \Gamma_i \vdash_{\mathcal{K}} P_i}{\Gamma_1 \odot \Gamma_2 \vdash_{\mathcal{K}} P_1 \mid P_2} \text{ (U-PAR)} \qquad \frac{\Gamma \vdash_{\mathcal{K}} P \qquad \Gamma(x) = \sigma}{(\boldsymbol{\nu} x)\, \Gamma \vdash_{\mathcal{K}} (\boldsymbol{\nu} x : \sigma)\, P} \text{ (U-RES)}$$

$$\frac{\begin{array}{c}\forall i : (\Sigma_i; \Xi_{\mathrm{L}i} \blacktriangleleft \Xi_{\mathrm{E}i}) \vdash_{\mathcal{K}} G_i.P_i \\ \Xi_{\mathrm{E}} \preceq \bigwedge_i \Xi_{\mathrm{E}i}\end{array}}{(\bigwedge_i \Sigma_i; \bigwedge_{k \in \mathcal{K}} \mathsf{sum}_k(\{p_i\}_i, \Xi_{\mathrm{E}}) \wedge \bigvee_i \Xi_{\mathrm{L}i} \blacktriangleleft \Xi_{\mathrm{E}}) \vdash_{\mathcal{K}} \sum_i G_i.P_i} \text{ (U-SUM)}$$

$$\frac{\Gamma \vdash_{\mathcal{K}} P \quad \mathsf{sub}(G) = p \quad \mathsf{obj}(G) = \tilde{x}}{\begin{array}{r}\left(p : \sigma; \blacktriangleleft p^m \wedge \bar{p}^{m'}\right) \odot \\ \left(; p^{\#(G)} \blacktriangleleft\right) \odot \\ !_{\text{if } \#(G) = \omega}\, (\boldsymbol{\nu} \mathsf{bn}(G)) \Big(\Gamma \odot \\ \overline{\sigma}[\tilde{x}] \odot \\ (; \bigwedge_{k \in \mathcal{K}} \mathsf{prop}_k(\sigma, G, m, m') \blacktriangleleft) \Big) \vdash_{\mathcal{K}} G.P\end{array}} \text{ (U-PRE)}$$

Type System (Existential)

$$\frac{\forall i : \Gamma_i \vdash_{\mathcal{K}} P_i}{\Gamma_1 \odot \Gamma_2 \vdash_{\mathcal{K}} P_1 \mid P_2}\ (\text{E-PAR}) \qquad \frac{\Gamma \vdash_{\mathcal{K}} P \qquad \Gamma(x) = \sigma}{(\boldsymbol{\nu} x)\,\Gamma \vdash_{\mathcal{K}} (\boldsymbol{\nu} x : \sigma)\, P}\ (\text{E-RES})$$

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Properties

- **A** — Activeness

$$\text{prop}_{\mathbf{A}}(G, \sigma, m, m') = \begin{cases} \text{sub}(G)_{\mathbf{A}} & \text{if } \#(G) = \omega \text{ or } m' \neq \star \\ \top & \text{otherwise} \end{cases}$$

- **R** — Responsiveness
- **D** — Determinism (Functionality)
- **I** — Isolation
- **df** — Lock-Freedom
- **N** — Non-Reachability
- ϖ — Termination

Properties

- **A** — Activeness
- **R** — Responsiveness

$$\mathrm{prop}_{\mathbf{R}}(\sigma, G, m, m') = \mathrm{sub}(G)_{\mathbf{R}} \lhd \begin{cases} \sigma[\mathrm{obj}(G)] & \text{if } G \text{ is an input} \\ \overline{\sigma}[\mathrm{obj}(G)] & \text{if } G \text{ is an output} \end{cases}$$

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- **A** — Activeness
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$$\varphi_{\mathbf{D}}(\sigma, G, m, m') \stackrel{\text{def}}{=} \begin{cases} \perp & \text{if } \star \in \{m, m'\} \text{ and } \omega \notin \{m, m'\} \\ \overline{\text{sub}(G)_{\mathbf{D}}} & \text{otherwise} \end{cases}$$

$$\varphi_{\mathbf{D}}(\{p_i\}_i, \Xi) \stackrel{\text{def}}{=} \begin{cases} \perp & \text{if } \Xi \text{ has concurrent environment } p_i \\ \top & \text{otherwise} \end{cases}$$

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$$\mathsf{prop}_{\mathbf{N}}(G, \sigma, m, m') \stackrel{\mathrm{def}}{=} \mathsf{sub}(G)_{\mathbf{N}} \lhd \perp \wedge \tau_{\mathbf{N}} \lhd \overline{\mathsf{sub}(G)}_{\mathbf{N}}$$

Universal Soundness

- Based on transition sequences?
  Semantic Predicates aren't transition based!
- Based on contextual semantics?
  "$\Delta_1 \lhd \Delta_2 \models P$ if $\forall Q$ s.t. $\Delta_2 \vdash Q$: $\Delta_1 \models P \mid Q$."
  The definition is circular!
- Implicit definition?
  "The set of correct typed processes is the largest that satisfies the above"
  There are many solutions!
- Stricter implicit definition?
  "The set of correct typed processes is the intersection of all those that satisfy the above"
  The intersection is empty!
- To be continued ...

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Existential Soundness

Structural Liveness Strategies

$$
\begin{aligned}
\rho &::= \pi\delta \quad | \quad \mathfrak{l} \quad | \quad \cdots \\
\delta &::= \div\ \rho \quad | \quad [\,s\,] \\
\pi &::= (\mathfrak{l}|\rho) \quad | \quad (\mathfrak{l}|\bullet) \quad | \quad (\bullet|\rho) \\
s &::= p_1 + p_2 + p_3 \ldots
\end{aligned}
$$

$\mathfrak{l}$: Guard reference

$\bullet$: Environment

$(\mathfrak{l}|\rho)$: Make $\mathfrak{l}$ and ρ communicate.

Future Work

- Generic Universal Soundness Proof
- Recursivity and **B**ounded Channels.
- Channel Type Reconstruction.
- Software Implementation.

Link to Appendices